Curvilinear integral

1) curvilinear integral - first kind

i) If **f (x, y, z) is** defined and constant function in each point of section the smooth curve **c**:

x=x(t)
y=y(t) where is $t_1 \le t \le t_2$, and ds- differential arch of curve
z=z(t)

then curvilinear integral is calculated by the formula:

$$\int_c f(x,y,z)ds = \int_{t_1}^{t_2} f[x(t),y(t),z(t)]\sqrt{(x_t^`)^2+(y_t^`)^2+(z_t^`)^2}\,dt$$

This integral does not depend on the orientation of the curve!

ii) If curve have form **c: y=y(x)** $a \le x \le b$ then, we have:

$$\int_c f(x,y)ds = \int_a^b f(x,y(x))\sqrt{1+(y_x^`)^2}\,dx$$

iii) If curve have form **c: x=x(y)** $m \le y \le n$ then, we have:

$$\int_c f(x,y)ds = \int_m^n f(x(y),y)\sqrt{1+(x_y^`)^2}\,dy$$

Calculate the length of curve c: **S =** $\int_c ds$

2. curvilinear integral – second kind

i) If the curve c is given with parametric equations:

x=x(t)
y=y(t) where is $t_0 \le t \le t_1$ then:
z=z(t)

$$\int_c P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz = \int_{t_0}^{t_1} [P(x(t),y(t),z(t))x_t^{`} + Q(x(t),y(t),z(t))y_t^{`} + R(x(t),y(t),z(t))z_t^{`}]dt$$

ii) If curve have form y=y(x) and $a \le x \le b$ then:

$$\int_c P(x,y)dx + Q(x,y)dy = \int_a^b [P(x,y(x)) + Q(x,y(x))y_x^{`}]dx$$

iii) If curve have form x=x(y) and $m \le y \le n$ then:

$$\int_c P(x,y)dx + Q(x,y)dy = \int_m^n [P(x(y),y)x_y^{`} + Q(x(y),y)]dy$$

Take heed: curvilinear integral – second kind depends on the orientation of the curve: $\int_{C^+} = -\int_{C^-}$

Positive direction is contrary to the direction of movement of clockwise on the clock.

Independence curvilinear integrals of the path of integration:

Following affirmations are equivalent:

1) $\int_C P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz$ does not depend on the path of integration

2) There are function u=u(x,y) so that : du = P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz and:
$\int_A^B$ P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz = u(B) - u(A)

3) $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$, $\frac{\partial P}{\partial z} = \frac{\partial R}{\partial x}$, $\frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y}$

4) $\int_C$ P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz = 0 if the curve c is closed.

Green's formula:

$$\oint_C P(x,y)dx + Q(x,y)dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dxdy$$

From the Green formula can be easily proved that the **surface of area A (D),** which is limited with curve C, formula:

$$A(D) = \frac{1}{2} \int_C xdy - ydx$$